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An experiment is proposed to visualize stroboscopically in real time the dynamics of a photon oscillating between two cavities. The visualization is implemented by a sequence of weak measurements (POVM), which are carried out by probing one of the cavities with a Rydberg atom and detecting a resulting phase shift by Ramsey interferometry. This way to measure the number of photons in a cavity was experimentally realized by Brune et al. . We suggest a feedback mechanism which minimizes the disturbance due to the measurement and enables a detection of the original evolution of the radiation field.

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In a preceeding paper [1] we have shown theoretically and numerically that it is feasible to monitor in real time a dynamical process occurring in a single two-level system with state vector

$$|\tilde{\psi}(t)\rangle = \tilde{c}_1(t)|\varphi_1\rangle + \tilde{c}_2(t)|\varphi_2\rangle \,. \tag{1}$$

Our aim in the following is to describe an experimental set up, which may possibly be used to demonstrate that the time behaviour as given by $|\tilde{c}_2(t)|^2$ can be registered while only weakly influencing the original dynamics of $|\tilde{\psi}(t)\rangle$.

How can this aim be achieved? Since projection measurements, which are also called sharp measurements, severely alter the original motion of $|\tilde{\psi}(t)\rangle$, they are not suitable in an one shot situation where only a single realization of this motion is available. One needs instead weak (or unsharp) measurements by which the state of the system is less disturbed but nevertheless some information about the state is provided. A single weak measurement can be realized by suitably entangling the two-level system with a quantum meter via a unitary transformation (premeasurement) followed by a projection measurement on the meter. This gives the measurement result, which is read off.

To track the development of $|\tilde{c}_2(t)|^2$ in time, a sequence of weak measurements is necessary. The corresponding series of measurement results can then be appropriately processed to give the final measurement readout. Two conditions may thus be fulfilled simultaneously: i.) The back action of the measurements does only moderately disturb the original dynamics of the system given by the evolution of $|\tilde{\psi}(t)\rangle$ and ii.) the variance of the measurement results is small enough to enable a reliable estimate of the original time behaviour of $|\tilde{c}_2(t)|^2$.

In this paper we sketch an experiment to visualize a known dynamics. This is meant to be a first step towards the tracking of an unknown motion of the state of a system with a finite dimensional Hilbert space. We mention that in this case QND measurements do not exist whereas weak measurements seem to represent a promising starting point [2].

The single weak measurement of the type in question belongs to the large class of generalized measurements in which observables are represented by positive operator valued measures (POVM). For a survey see [3]. These measurements are usually studied with respect to single joint measurements of incompatible observables. In contrast to this we deal with a sequence of fixed weak measurements of the same observable and demonstrate the use of POVM in this context. We mention that the results obtained for truly continuous measurements are in our stroboscopical situation of limited use.

A series of weak measurements was employed to carry out a QND measurement of small photon numbers in an experiment of Brune, Haroche et al. [4], which was theoretically analyzed in [5,6] and experimentally realized in [7]. While the weakness of the measurements has been considered as an obstacle there, it turns out to be an advantage when it comes to the detection of dynamics. The experimental setup we sketch in the following is based on the Brune-Haroche experiment. We have added a feedback mechanism, which was necessary to decrease the back action of the measurements.

We first describe the quantum system and its undisturbed dynamics which we want to visualize. The system consists of one photon with frequency ω shared by two equally constructed, coupled cavities $\mathcal{C}_1$ and $\mathcal{C}_2$. One could think of two identical cavities connected by a waveguide (cf. [8,9]) or, to use an idealized picture, two cavities separated by a transmissive mirror. The cavities are assumed to have infinite damping time. Their coupling is modeled by the interaction Hamiltonian of the Jaynes-Cummings type:

$$H = \hbar g (a_1 a_2^\dagger + a_1^\dagger a_2) \tag{2}$$

with coupling constant g. In the interaction picture, which we are going to use, (2) is the full Hamiltonian. The indices refer to the cavity numbers. Such a coupling of two cavities has also been considered by Zoubi et al. in [10]. The photon which is delocalized over the two cavities can be described as a superposition of two states:

$$|\tilde{\psi}(t)\rangle = \tilde{c}_1(t)\,|1,0\rangle + \tilde{c}_2(t)\,|0,1\rangle \,. \tag{3}$$

The first and the second slot in the ket represent the number of photons in cavity $\mathcal{C}_1$ and cavity $\mathcal{C}_2$ respectively. For the initial state $|\tilde{\psi}(t=0)\rangle = |1,0\rangle$ we find Rabi-oscillations with the Rabi-frequency $\Omega_R := 2g$

$$|\tilde{c}_2(t)|^2 = \sin^2(gt) \,. \tag{4}$$

It is our goal to measure this original evolution of $|\tilde{c}_2(t)|^2$ in real time by probing the coupled cavities with atoms. For this purpose we first turn to the premeasurement. We sent a Rydberg atom with three effective energy levels g, e, i and velocity v through the first cavity $\mathcal{C}_1$ (cp. [4,5]). The passage time L_c/v (L_c is the cavity length) is assumed to be much shorter than the period $T_R := 2\pi/\Omega_R = \pi/g$ of the oscillations of $|\tilde{c}_2(t)|^2$. Then the coupling of the two cavities is negligible during the time the atom spends in the cavity. The detuning of the atomic transitions with respect to the frequency of the cavity mode ω is such that the interaction between the atom and $\mathcal{C}_1$ is dispersive and only the energy levels e and i suffer an appreciable dynamical stark shift. Provided the atom enters the cavity in in a superposition of states $|g\rangle$ and $|e\rangle$, the effective Hamiltonian reads (cp. eqn. (16) in [5]):

$$H_{\text{int}} = \frac{\hbar\Omega^2}{\delta}\,|e\rangle\langle e| \otimes a_1^\dagger a_1 \,, \tag{5}$$

where $\delta := \omega - \omega_{ie}$ and $\Omega = \overline{\Omega(r)}$ is the Rabi frequency averaged over the path of the atom through the cavity. With (5) the state of the enlarged system composed of the atom and the photon field changes according to

$$\begin{aligned}(c_e|e\rangle + c_g|g\rangle) \otimes |\psi\rangle \rightarrow c_e|e\rangle \otimes U_{\mathcal{C}_1}|\psi\rangle \\ +c_g|g\rangle \otimes |\psi\rangle\end{aligned} \tag{6}$$

with $U_{\mathcal{C}_1}$ being diagonal in the basis $|1,0\rangle$ and $|0,1\rangle$:

$$U_{\mathcal{C}_1} := e^{-i\varepsilon_1}|1,0\rangle\langle 0,1| + |0,1\rangle\langle 1,0| \,, \tag{7}$$

and $\varepsilon_1 := \frac{\hbar\Omega^2}{\delta}\frac{L_{\mathcal{C}}}{v}$. $|\psi\rangle$ represents the state of the 1-photon-field probed by atoms. The net effect of the atom-field coupling described by the interaction Hamiltonian (5) is that only the amplitude of the alternative $|e\rangle\otimes|1,0\rangle$ suffers a phase shift $e^{-i\varepsilon_1}$ while the amplitudes of the other quantum alternatives remain unchanged.

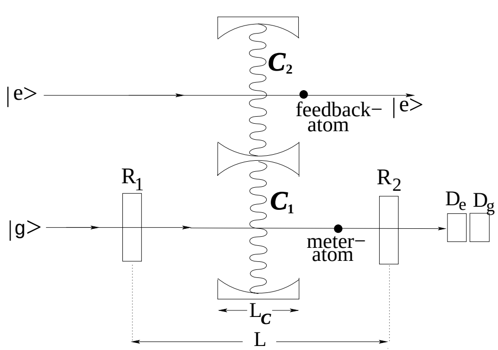

FIG. 1. Experimental setup

Phase shifts between several quantum alternatives may be measured by interferometry. As proposed in [4] it is convenient to use the Ramsey method of separated oscillatory fields (see Fig. 1). To this end a Rydberg atom is initially prepared in state $|g\rangle$. Before entering cavity $\mathcal{C}_1$ the state of the atom is tranformed into a superposition of states $|e\rangle$ and $|g\rangle$ by a first classical oscillatory microwave field R_1 with frequency ω_r. In the cavity $\mathcal{C}_1$ the atomic state becomes entangled with the state of the cavities as discussed above. After leaving $\mathcal{C}_1$ the atom crosses a second classical microwave field R_2 which is in phase with R_1 and positioned at the distance L from it. The total state change of duration $\delta\tau$ amounts to (cf. [5]) $|\Psi(t_0)\rangle \rightarrow |\Psi(t_0+\delta\tau)\rangle$, where the product state before the atom enters $\mathcal{C}_1$ is given by

$$|\Psi(t_0)\rangle = |g\rangle \otimes |\psi(t_0)\rangle = |g\rangle \otimes \big(c_1(t_0)|1,0\rangle + c_2(t_0)|0,1\rangle\big) \tag{8}$$

and the final entangled state equals

$$\begin{aligned}|\Psi(t_0+\delta\tau)\rangle = |e\rangle \otimes \big(u_1^e c_1(t_0)|1,0\rangle + u_2^e c_2(t_0)|0,1\rangle\big) \\ + |g\rangle \otimes \big(u_1^g c_1(t_0)|1,0\rangle + u_2^g c_2(t_0)|0,1\rangle\big) \,.\end{aligned} \tag{9}$$

This completes the premeasurement. The coefficients in (9) are given by

$$u_1^e = \frac{1}{2}\sin\left(\frac{\pi}{2}\frac{v_0}{v}\right)\left[e^{i(\varphi_0-\varepsilon)\frac{v_0}{v}} + 1\right] \tag{10}$$

$$u_1^g = \cos^2\left(\frac{\pi}{4}\frac{v_0}{v}\right) - \sin^2\left(\frac{\pi}{4}\frac{v_0}{v}\right) e^{i(\varphi_0-\varepsilon)\frac{v_0}{v}} \tag{11}$$

and $u_2^e = u_1^e(\varepsilon=0)$, $u_2^g = u_1^g(\varepsilon=0)$ with $\varepsilon = \frac{\hbar\Omega^2}{\delta}\frac{L_{\mathcal{C}}}{v_0}$. v_0 characterizes the Ramsey fields and depends on the length l_r of each of the corresponding cavities and the effective Rabi-frequency Ω_r inside these cavities: $v_0 := 2l_r\,\Omega_r\,/\pi$. $\varphi_0 := (\omega_r - \omega_{eg})\frac{L}{v_0}$ is the phase shift which is induced by the Ramsey cavities in the case $v = v_0$. An analogous result was obtained in eqn. (A7) of [5] for the initial atomic state being $|e\rangle$. Eqn. (9) shows that the meter states $|e\rangle$ and $|g\rangle$ couple in general to both cavity states $|1,0\rangle$ and $|0,1\rangle$. This is a characteristic trait of a weak measurement.

After the atom has left the second Ramsey field R_2 its energy is finally detected in a projection measurement by field ionization counters D_e and D_g. The state of the composite system after a measurement with result $l \in \{e,g\}$ reads $|\Psi_l(t_0+\delta\tau)\rangle = |l\rangle \otimes |\psi_l(t_0+\delta\tau)\rangle$ with photon state

$$\begin{aligned}|\psi_l(t_0+\delta\tau)\rangle = |u_1^l|\,c_1(t_0)|1,0\rangle \\ +|u_2^l|\,e^{i(\chi_2^l-\chi_1^l)}c_2(t_0)|0,1\rangle \,.\end{aligned} \tag{12}$$

and $u_j^l = |u_j^l|e^{i\chi_j^l}$ for $j \in \{1,2\}$. Here a global phase factor has been omitted. The probability to obtain the related measurement result l is given by the expectation value of the corresponding projector: $\text{prob}(l) = \langle\, (\,|l\rangle\langle l| \otimes \mathbb{1}\,)\,\rangle_{\Psi(t_0+\delta\tau)}$. Eqn. (12) shows that after the measurement the photon is in general not localized in one of the cavities. The disturbance of the photon state due to the measurement may be small. Because of the Rabi-evolution between the measurements this set up represents no QND measurement of the photon number as it has been in the Brune-Haroche experiment.

Refering to the photon field only, the change of its state due to a single measurement with result l can be expressed by an operation M_l: $|\psi(t_0)\rangle\rangle \to |\psi_l(t_0+\delta\tau)\rangle\rangle = M_l|\psi(t_0)\rangle\rangle$. Like all bounded operators, M_l can be written as "phase" times "modulus" (polar decomposition)

$$M_l = U_l|M_l| \tag{13}$$

with unitary transformation

$$U_l := |1,0\rangle\langle 0,1| + e^{i(\chi_2^l - \chi_1^l)}|0,1\rangle\langle 1,0| \tag{14}$$

and positive operator

$$|M_l| := |u_1^l|\,|1,0\rangle\langle 0,1| + |u_2^l|\,|0,1\rangle\langle 1,0|\ . \tag{15}$$

The probability to obtain the outcome l is then:

$$\text{prob}(l) = \langle\, M_l^\dagger M_l\,\rangle_{\psi(t_0)} = \langle\, |M_l|^2\,\rangle_{\psi(t_0)}\ . \tag{16}$$

$E_l := |M_l|^2$ is also called effect. In this way we obtain e.g. for the probability to measure the energy e: $p_e = p_1|c_1|^2 + p_2|c_2|^2$, where $p_j := |u_j^e|^2$ is fixed by (10) and (11).

The effects have the property $E_e + E_g = \mathbb{1}$ and generate a positive operator valued measure (POVM). In the special case where $u_1^e = u_2^g = 1$ and $u_2^e = u_1^g = 0$, the operation $M_l = E_l$ is a projector. If on the other hand $E_l = \mathbb{1}$, no measurement at all has taken place but only an unitary development ($M_l = U_l$). These two cases are the extremes of a sharp and a totally unsharp measurement. By varying the parameters $v\,, v_0\,, \varphi_0$ and ε of the setup all degrees of "weakness" between these two extreme cases as well as the extremes themselves can be reached.

Eqn. (16) shows that the information obtained by the generalized measurement is solely contained in $|M_l|$. This part of the operation M_l in (13) represents at the same time the unavoidable minimal disturbance of the system by the measurement (cp. [11]). But our set up causes in addition by means of U_l a purely unitary or Hamiltonian evolution of the state, which modifies the photon state without being necessary for the extraction of information. Since we want to disturb the original state motion as little as possible, we have to install a Hamiltonian feedback mechanism which compensates U_l given by (14). This can be done by supplementing the set up as follows:

After a measurement beginning at an arbitrary time $t = t_0$ with outcome l an atom prepared in state $|e\rangle$ is sent through the second cavity $\mathcal{C}_2$. As in the case where an atom crosses cavity $\mathcal{C}_1$ the unitary evolution is again governed by the dynamical Stark effect, with the only difference that now the energy shift depends on the number of photons in $\mathcal{C}_2$ instead of $\mathcal{C}_1$:

$$|e\rangle \otimes |\psi_l(t_0+\delta\tau)\rangle\rangle \to |e\rangle \otimes U_{\mathcal{C}_2}|\psi_l(t_0+\delta\tau)\rangle\rangle\,, \tag{17}$$

with

$$U_{\mathcal{C}_2} := |1,0\rangle\langle 0,1| + e^{-i\varepsilon_2}|0,1\rangle\langle 1,0|\ . \tag{18}$$

The combined influence of the measurement and feedback leads to

$$U_{\mathcal{C}_2}|\psi_l(t_0+\delta\tau)\rangle\rangle = U_{\mathcal{C}_2}U_l|M_l|\,|\psi(t_0)\rangle\rangle\ . \tag{19}$$

The condition for compensation of U_l in (14) is therefore $U_{\mathcal{C}_2}U_l = \mathbb{1} \Leftrightarrow \varepsilon_2 = \chi_2^l - \chi_1^l$, where χ_j^l may be obtained from (10) and (11). This condition demands that the compensating phase $\varepsilon_2 = \frac{\hbar\Omega^2}{\delta_f}\frac{L_{\mathcal{C}}}{v_f}$ (f denotes the feedback) has to be chosen depending on the measurement outcome l. We see two ways to vary ε_2. One is to select an appropriate velocity v_f of the feedback atom sent through the upper cavity $\mathcal{C}_2$. The other possibility consists in setting up a suitable detuning δ_f. This can be done by shifting the atomic energies by means of an static electric field in the cavity $\mathcal{C}_2$ cp. [12]. Please note that it makes no difference whether the atom sent through cavity $\mathcal{C}_2$ is thereafter measured or not because the composite system after the interaction is in a product state.

In order to reach our final aim of monitoring the original Rabi-oscillations of $|\tilde{c}_2(t)|^2$ as good as possible, a sequence of measurements at times $t_n = n\tau$ with $n = 1, 2, 3\ldots$ has to be carried out. Between two consecutive measurements the system evolves undisturbed according to the Hamiltonian (2). The resulting total evolution of the system is given by $c_2(t)$ instead of $\tilde{c}_2(t)$. We now describe how to process the data obtained in the single measurements in order to extract information about $|c_2(t)|^2$. First of all we divide the sequence of results with values e and g into groups of N. From each so called "N-series" we extract the relative frequency $r := N_e/N$ of the number N_e of e-results. It has been shown in [1] that by means of r a best guess of $|c_2(t_0)|^2$ at time t_0 when the first measurement of the respective N-series began can be obtained according to

$$\text{BG}_2(t_0) = \frac{r(t_0) - p_1}{\Delta p} \tag{20}$$

with $\Delta p := p_2 - p_1 = |u_2^e|^2 - |u_1^e|^2$ of (10) and (11). $\text{BG}_2(t_0)$ may be negative. This estimation of $|c_2(t_0)|^2$ can be good only if the duration of the N-series $N\tau$ is much smaller than the period T_R of the oscillations of the system. The sequence of BG_2 at various times serves as the final readout of the sequential measurement.

We have two competing influences on the system: The strength of the original dynamics is proportional to g or T_R^{-1}. The measurements on the other hand hinder this dynamics the more the stronger they are and the quicker they are repeated. The Zeno effect demonstrates this! General arguments given in [1] and numerical calculations showed that for our purpose a favorable balance of the influences is obtained if the so called fuzziness $F := 4\pi \frac{p_0(1-p_0)}{(\Delta p)^2} \frac{\tau}{T_R}$ with $p_0 := (p_1 + p_2)/2$ is adjusted to be close to one: $F \approx 1$. We choose the experimental parameters ε, φ_0, v_0, v and τ correspondingly.

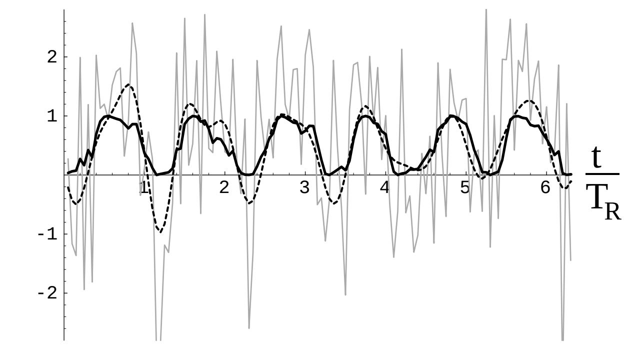

FIG. 2. Under a sequence of appropriate weak measurements the measurement readout BG_2 (grey curve) is correlated with the state evolution $|c_2(t)|^2$ (black curve). This becomes evident after noise reduction (dashed curve) of the readout BG_2, which was carried out taking into account approximately 12 Rabi-cycles. Parameter values: $\frac{v}{v_0} \in [1 - 0.08, 1 + 0.08]$, $\varepsilon = 0.068\pi$, $\varphi_0 = \pi$, $\tau = 0.002 T_R$ and $N = 25$.

In the following we discuss our results. We have simulated numerically all the processes described above including the feedback. In a realistic experiment the velocity v of the probing atoms will vary from one single measurement to the other. We have accordingly tolerated the velocities v to fluctuate uniformly by $\pm 10\%$ about the desired mean value. The resulting dynamics of the state under the influence of stroboscopically applied weak measurements is given by the $|c_2(t)|^2$-curve (black) in Fig. 2. The measurement readout which is defined as best guess $BG_2(t)$ of $|c_2(t)|^2$ (grey curve) has been further processed to the noise reduced BG_2-curve (dashed). We find a high correlation including the phase between the noise reduced BG_2-curve and the $|c_2|^2$-curve. The actual evolution of the state is therefore well monitored in time. The $|c_2|^2$-curve reflects the fact that the original Rabi-oscillations have been disturbed by the measurement, though they are only slightly modified. Fig. 3 shows the powerspectrum of the $|c_2|^2$-curve (black) and the measurement readout BG_2 (grey). Both curves are peaked at the Rabi frequency Ω_R.

To sum up: The original Rabi-oscillations are well tracked in phase and frequency. We regard this result as a first step towards the visualization of unknown motion of a state in real time.

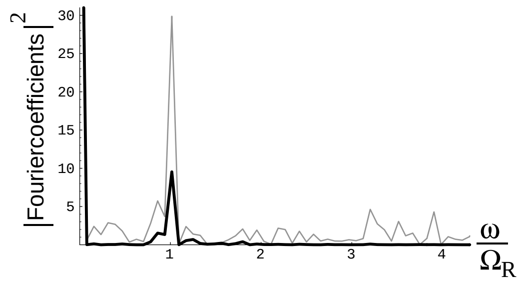

FIG. 3. Powerspectra of $|c_2(t)|^2$ (black)

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